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LETTER TO THE EDITOR

Spiral self-avoiding walks

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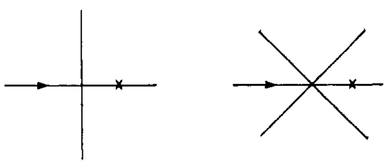
Abstract. Self-avoiding walks with a 'spiral' constraint, on the square lattice, are enumerated up to 40 steps. Numerical evidence suggests that they belong to a new universality class.

Recently, there has been an interest in the problem of self-avoiding walks (SAW) in 2D (see reviews by Flory 1969, McKenzie 1976 and de Gennes 1979). Exact values of critical exponents were conjectured (Nienhuis 1982) and new extensive enumerations, Monte Carlo and RG studies were reported (Le Guillou and Zinn-Justin 1980, Derrida 1981, Kolb *et al* 1982, Guttmann 1983, Majid *et al* 1983, Havlin and Ben-Avraham 1983). In particular, attempts have been made to parametrise corrections to the leading critical behaviour (Grassberger 1982, Nienhuis 1982, Adler 1983, Djordjevic *et al* 1983, Guttmann 1983, Privman 1983). A related problem of 'directed' SAW (Fisher and Sykes 1959) has been studied recently (Cardy 1983, Redner and Majid 1983, Szpilka 1983). Similarly to percolation (see a review by Kinzel (1983)), the directed SAW problem is in a new universality class which corresponds to a 'trivial' 1D-type critical behaviour.

Grassberger (1982) enumerated SAW with constrained steps, on the square and the triangular lattices: a random walker is forced to change direction at each step, as illustrated schematically by



where a step to a neighbour in the direction of the preceding step (→) is forbidden (—×—) but all other paths are allowed. Such 'microscopic' constraints do not change the universality class: on a macroscopic scale the pattern of wandering of the random walker is not affected. Numerically (Grassberger 1982) the critical exponent ν is found to be consistent with the unconstrained problem value.

We consider here a different microscopic constraint: a step which points 90° clockwise with respect to the preceding step, on the square lattice, is forbidden:

Obviously, typical long walks will wind counterclockwise around the origin: the microscopic constraint causes a new macroscopic pattern of behaviour, thus such 'spiral' SAW are expected to belong to a new universality class. In table 1 we list results of enumeration of walks of up to 40 steps (bonds), with *the first step fixed* (say $\rightarrow$). We *assumed* that the total number of N-step walks, c_N, and the mean-square end-to-end distance of the N-step walks, $\rho_N \equiv \langle R_N^2 \rangle$, behave similarly to the unconstrained problem:

$$c_n \approx \text{constant}\, \mu^n n^{\gamma-1} \qquad \text{and} \qquad \rho_n \approx \text{constant}\, n^{2\nu}$$

but with new exponents γ and ν. We attempted to estimate μ, γ and ν using conventional series analysis techniques.

Table 1. The values of c_N and $c_N\rho_N$ for 'spiral' SAW ($N \leq 40$).

N	c_N	$c_N\rho_N$	N	c_N	$c_N\rho_N$
1	1	1	21	8 962	316 882
2	2	6	22	12 329	461 054
3	4	20	23	17 019	665 803
4	7	52	24	23 169	954 540
5	13	117	25	31 589	1 359 253
6	21	240	26	42 599	1 923 224
7	37	461	27	57 453	2 704 453
8	57	844	28	76 796	3 780 892
9	95	1 487	29	102 588	5 256 708
10	143	2 548	30	136 019	7 269 478
11	227	4 251	31	180 131	10 002 115
12	335	6 960	32	237 061	13 695 304
13	513	11 185	33	311 489	18 664 481
14	744	17 702	34	407 097	25 322 744
15	1106	27 626	35	531 113	34 209 401
16	1580	42 584	36	689 678	46 022 352
17	2294	64 878	37	893 884	61 668 524
18	3232	97 820	38	1153 837	82 317 142
19	4600	146 032	39	1486 445	109 472 221
20	6402	216 048	40	1908 002	145 064 884

Padé analysis gave less stable results than ratio techniques. The ratio-type analyses reveal complicated patterns of behaviour of approximants to μ, γ and ν: there are strong fluctuations of period 2, due to 'antiferromagnetic' singularities in the generating functions for both c_N and ρ_N. After cancelling these oscillations: using even–even and odd–odd ratios, etc, the sequences still have complicated pattern of convergence which is slow, with residual irregular longer-period fluctuations superimposed. We will not present details of the (standard) analyses, but only the resulting estimates:

$$\mu = 1.15 \pm 0.15, \qquad \gamma = 5.2 \pm 1.3, \qquad \nu = 0.62 \pm 0.06.$$

These ranges may still possess systematic errors, due to the above-mentioned irregular convergence of the series for $N \leq 40$; however, the exponent estimates are well away from the usual 2D SAW values ($\nu = 0.75$ and $\gamma = 1.343\,75$), confirming that the 'spiral' SAW belong to a new universality class.

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References

Adler J 1983 *Preprint*
Cardy J L 1983 *J. Phys. A: Math. Gen.* **16** L355
Derrida B 1981 *J. Phys. A: Math. Gen.* **14** L5
Djordjevic Z V, Majid I, Stanley H E and Dos Santos R J 1983 *J. Phys. A: Math. Gen.* **16** L519
Fisher M E and Sykes M F 1959 *Phys. Rev.* **114** 45
Flory P J 1969 *Statistical Mechanics of Chain Molecules* (New York: Interscience)
de Gennes P G 1979 *Scaling Concepts in Polymer Physics* (Ithaca, NY: Cornell)
Grassberger P 1982 *Z. Phys.* B **48** 255
Guttmann A J 1983 *Preprint*
Havlin S and Ben-Avraham D 1983 *Phys. Rev.* A **27** 2759
Kinzel W 1983 in *Percolation Structures and Processes, Ann. IPS* vol 5, ed G Deutscher, R Zallen and J Adler (Bristol: Adam Hilger) p 425
Kolb M, Jullien R and Pfeuty P 1982 *J. Phys. A: Math. Gen.* **15** 3799
Le Guillou J G and Zinn-Justin J 1980 *Phys. Rev.* B **21** 3976
McKenzie D S 1976 *Phys. Rep.* **27C** 35
Majid I, Djordjevic Z V and Stanley H E 1983 *Phys. Rev. Lett.* **51** 143
Nienhuis B 1982 *Phys. Rev. Lett.* **49** 1062
Privman V 1983 *Physica* A in press
Redner S and Majid I 1983 *J. Phys. A: Math. Gen.* **16** L307
Szpilka A M 1983 *J. Phys. A: Math. Gen.* **16** 2883